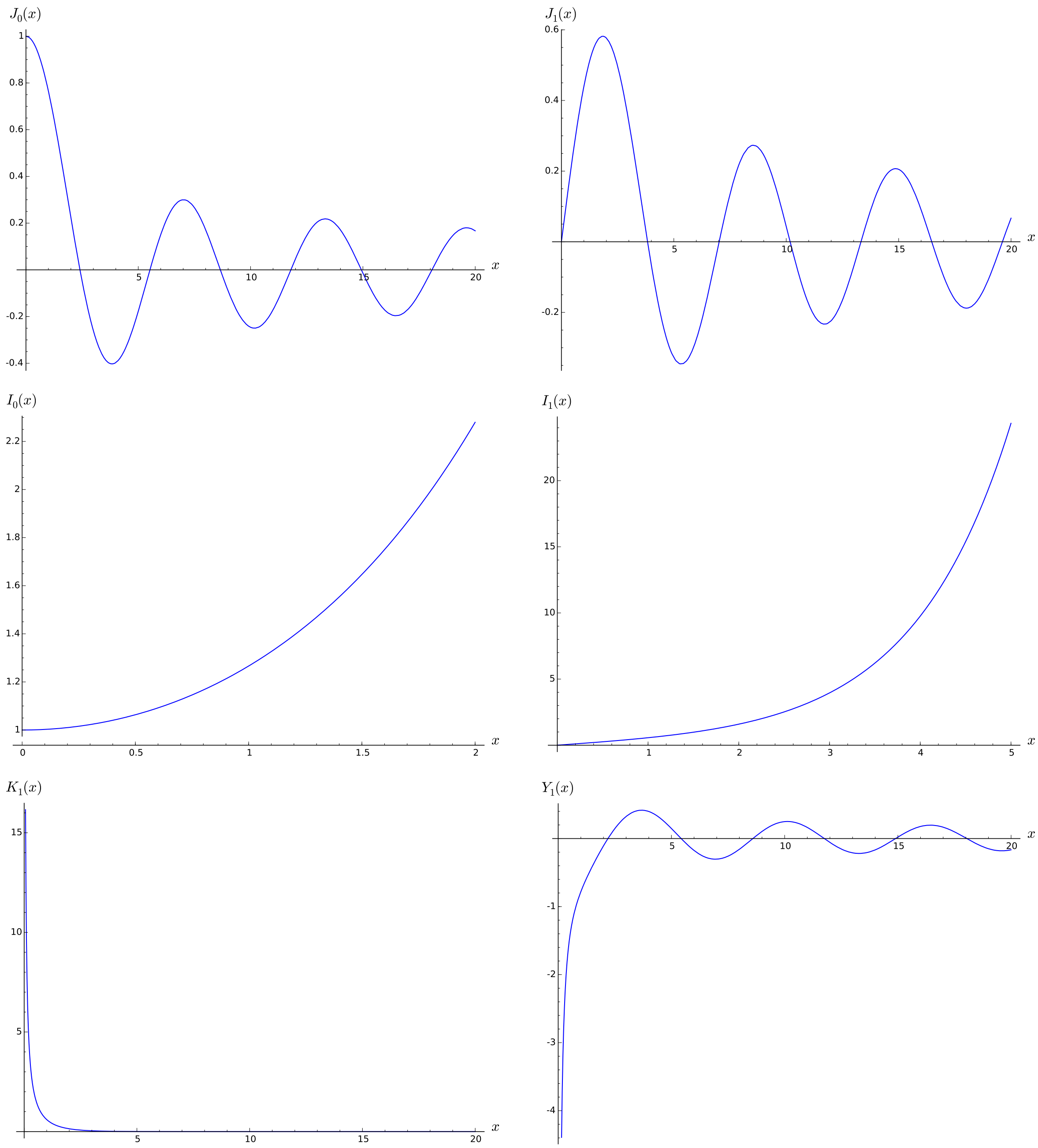
$$J_\nu(x) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{\nu+2m}}{2^{\nu+2m}\, m!\, \Gamma(\nu+m+1)}$$

is a *Bessel function of the first kind of order* ν. The general solution of $x^2\, y'' + x\, y' + (x^2 - \nu^2)\, y = 0$ is $y = c_1\, J_\nu(x) + c_2\, J_{-\nu}(x)$. If $\nu = n$ is an integer, the general solution is $y = c_1\, J_n(x) + c_2\, Y_n(x)$ where $Y_n(x)$ is the *Bessel function of the second kind of order* n. Here, $Y_n(x)$ equals $\frac{2}{\pi}\, \ln\left(\frac{x}{s}\right)$ plus a power series.

The solutions of $x^2\, y'' + x\, y' + (-x^2 - \nu^2)\, y = 0$ are expressible in terms of *modified Bessel functions of the first/second kind of order* ν, namely $I_\nu(x)$ and $K_\nu(x)$.

The graphs:



If $(1-a)^2 \geq 4c$ and if neither d, p nor q is zero, then, except in the obvious special case when it reduces to the Cauchy-Euler equation ($x^2y'' + axy' + cy = 0$), the differential equation

$$x^2y'' + x(a + 2bx^p)y' + [c + dx^{2q} + b(a + p - 1)x^p + b^2x^{2p}]y = 0$$

has as general solution

$$y = x^{\alpha}\, e^{-\beta x^p}[C_1\, J_\nu(\varepsilon x^q) + C_2 Y_\nu(\varepsilon x^q)]$$

where

$$\alpha = \frac{1-a}{2}, \qquad \beta = \frac{b}{p}, \qquad \varepsilon = \frac{\sqrt{|d|}}{q}, \qquad \nu = \frac{\sqrt{(1-a)^2 - 4c}}{2q}.$$

If $d < 0$, then J_ν and Y_ν are to be replaced by I_ν and K_ν, respectively. If ν is not an integer, then Y_ν and K_ν can be replaced by $J_{-\nu}$ and $I_{-\nu}$ if desired.